Engaging Activities for Middle School Math

An Honors Thesis (Honors 499)

By

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In addition, thank you to Dr. Kathryn Shafer, professor of mathematics at Ball State University, for helping me put together my research presentation to the Ball Sate Council of Teachers of Mathematics. It was through that presentation that much of this website began to come together.

Engaging Activities for Middle School Math

http://web.me.com/ldobson2/Middle_School_Math_Activities

Abstract

This project consists of four middle school lesson plans covering national mathematics standards for the middle school grades. The four topics will be pattern recognition, modular numbers, common factors, and prime numbers. By these activities, students are able to see how their seemingly tedious math curriculum has applications in daily life now and throughout history. These activities are presented on a website, so they are easily accessible to teachers everywhere.

Artist Statement

Rationale for the Project

During the fall semester of 2008, I was enrolled in the course MATHS 393: Teaching and Learning Mathematics in the Middle School. For the first time in my major, I was shown the components of a successful, meaningful lesson plan. More than how to simply create and write a lesson plan, I began to truly understand what meaningful mathematics looks like in a classroom. Meaningful mathematical activities guide students to move deeper beyond mathematical understanding than simply memorizing a formula or procedure. These activities point students toward the true meaning of math, which is problem solving and logic. Problem-based lessons and activities help students to see the importance of their work and move students from thinking math is simply a grade to realizing the importance of math in their daily lives.

It is critical to make the study of mathematics not only relevant and appropriately challenging, but to make it engaging and, simply put, fun. By grabbing students' interest through several simple activities and games that are placed on the website I created, students will be more interested in the topic of study. Once their attention is caught and their minds engaged, the lesson can be taught with enthusiasm and then easily led into extension activities.

In a typical school year, many middle school students will often ask the question, "Why do we need to know this?" Some teachers find it acceptable to merely answer the question with a canned response. However, my pre-service training has lead me to believe that it is important to not only tell students why math is important, but to show them why mathematics is an extremely meaningful and relevant area of study.

Cryptography, the practice of hiding information, is one modern area where mathematical concepts are heavily relied upon. The areas that I examined were pattern recognition, prime factorization, modular numbers, and greatest common factors. These are all topics that are covered in a typical middle school classroom, and cover several national academic standards set forth by the National Council of Teachers of Mathematics. I chose these four areas to research because they aligned nicely to the mathematical standards, they used developmentally-appropriate mathematics for middle school, and involved fascinating historical connections. Each of these four topics is aligned to a particular kind of cryptography and used for a specific purpose. For instance, RSA cryptography utilizes the concept of prime factorization. This sort of public-key cryptography is what keeps information on the internet safe from hackers. By showing students that prime factorization has a purpose, students will begin to see math beyond a simple question and answer. They will start to see math as an ever-present, important part of our lives, which makes our world function the way it does. Hopefully through these activities, students can see how math is much more than homework or tests, but is applicable and valuable.

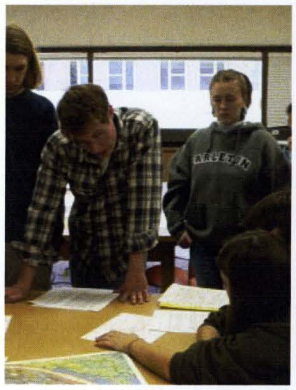
In presenting my research and lessons to the Ball State Council of Teachers of Mathematics, I hoped to give ideas to my fellow pre-service teachers. The vision was that my peers would be able to take my research and work and be able to use it in their future classroom to benefit their students and push their students to understand a deeper meaning of mathematics. With that same thought in mind, I created a website with the lessons and necessary worksheets and tables. This way, this information on cryptography in middle school math is accessible to anyone who would like to use it.

There is also a link on this website to different online activities for middle school math. I find it extremely crucial to make every single lesson as engaging as possible for students. Therefore, it is always helpful to have a bank of websites with good ideas, games, and activities. I have found these websites during my lesson planning and student teaching and want to also share these finds with anyone who stumbles upon my website.

My purpose of this project was to ultimately create a resource for middle school teachers to assist with lesson planning. The motivation activities listed under each topic can be a great springboard for any middle school classroom. The extension activities may be helpful in a gifted and talented classroom, or in a class that is interested in seeing the deeper meaning of mathematics. With this website, the lesson plans are already created and therefore don't require as much work from classroom teachers. By already doing the research and providing the information, I hope to have give middle school teachers a creative resource to spark student interest in the ever-practical field of mathematics.

MIDDLE SCHOOL MATHEMATICS

This site contains several ideas for engaging activities for middle school math. This site provides a few ideas on how to implement fun activities while still covering the necessary standards.



Activities for Middle School Math

Navigating the Site:

Along the top of the site are several mathematical topics covered in middle school math curriculum. Click on each link to get an idea on how to apply these concepts to cryptography.

If you have any questions or comments, please contact Lisa Dobson via email.

Greatest Common Factors

Target Audience:

Advanced middle school math students, grades 6-8. This activity could also be appropriate for a high school classroom.

National Standards (Grades 6-8):

Number and Operation: Develop an understanding of large numbers and recognize and appropriately use exponential, scientific, and calculator notation.
Number and Operation: Use factors, multiples, prime factorization, and relatively prime numbers to solve problems
Algebra: Analyze and generalize a variety of patterns
Algebra: Relate and compare different forms of representations of a relationship
Algebra: Model and solve contextualized problems using tables and equations
Reasoning and Proof: Examine patterns and structures to detect regularities
Reasoning and Proof: Formulate conjectures about observed regularities
Reasoning and Proof: Evaluate conjectures

Learning Objectives:

Students will be able to see how greatest common factors are used in a real life application. Students will also work through questions using their problem solving abilities.

Motivational Activities: Greatest Common Factors

- What is factoring? Students may have a difficult time fully grasping the concept of factors, and what it means for different sets of numbers to be factors of the same number.
- There is an NCTM: Illuminations online applet, Factorize, that uses graphing paper to display what it means for different pairs of numbers to be factors
- If students still need help and need to practice their factors, here is a fun "cop game" that helps students with memorizing factors and an NCTM: Illuminations two-person factor game

Number Cop NCTM: Illuminations Factor Game

Application Activity: The Vigenere Cipher

Simple substitution ciphers are easily broken because of their one-to-one correspondence with letters. The Vigenere Cipher is a polyalphabetic cipher, and therefore does not have this problem. This means the Vigenere Ciphers are much more difficult to crack. In fact, for a long period of time in history, the Vigenere cipher was thought to be unbreakable. However, we will prove that with our knowledge of mathematics, we are capable of cracking the cipher "impossible of translation." First, let's go over how the cipher works.

- These ciphers use a keyword. For this example, we will use the keyword "math"
- This is how the message "Rome is the greatest empire" would look

  Keyword: MATH MA THM ATHMATHM ATHMAT.
  Plaintext: ROME IS THE GREATEST EMPIRE.

- On the tables, there are Vigenere Grids. This is our tool to help us encode our message.
- The first letter in plaintext is R, and the first letter in the keyword is M. On the grid, find the plaintext letter R in the top row, and find the keyword letter M in the left column. The encoded message then uses the intersection of this row and column. The intersection is D, so the first letter of the encoded message is D.
- Continue on in this fashion until the entire message is coded.

Some questions to consider (some are questions for students, others are for you as a teacher to figure out):

- If you provided someone with the ciphertext, how could they decode it?
- What was the purpose of the keyword?
- To decode this cipher, an inverse process is used. What does inverse mean? Where have you encountered the use of an inverse before? Provide an example.
- When using the Caesar cipher, you found that there are 25 possible shifts. Therefore, a code using a Caesar cipher could be broken by trying all 25 shifts. How many arrangements would a code breaker have to try if he knew that a message was encoded using a keyword with two letters?
- Use your results from the previous question to determine the number of possible arrangements if SUPER were used as the keyword.
- Is it possible for a Vigenere Cipher to be cracked? Why or why not?
- The top of this sheet says this uses common factors. How and when are they used with the Vigenere Cipher?

  Clue: In long texts, there are often "strings" of letters that are repeated.

Make your own message, and using your own keyword, encode it with the Vigenere Cipher. Give the message to someone to try to decode (you can decide if you want to give them the keyword or not, but it is much more friendly to give it to them).

Obviously, it is much easier to have a computer do the work for this cipher (It is really easy to get weighed down by the computation and meticulous charts. It may be difficult to see the bigger picture for some students)

- You can use an online resource to possibly aid some students with seeing what is happening: Cryptoclub
- Also, this is a quick way to encode and decode messages

Necessary Worksheets:

Vigenere Grid

Sources Used:

National Council of Teachers of Mathematics: Illuminations

NCTM: Illuminations Factorize

Number Cop: Hotmath.com

NCTM: Illuminations factor game

The Cryptoclub

Made on a Mac

Prime Numbers

Target Audience:

Advanced middle school math students, grade 8. This activity could also be appropriate for a high school algebra classroom.

National Standards (Grades 6-8):

Number and Operation: Develop an understanding of large numbers and recognize and appropriately use exponential, scientific, and calculator notation.
Number and Operation: Use factors, multiples, prime factorization, and relatively prime numbers to solve problems
Numbers and Operation: Develop and analyze algorithms for computing with integers
Algebra: Analyze and generalize a variety of patterns
Reasoning and Proof: Examine patterns and structures to detect regularities
Reasoning and Proof: Formulate conjectures about observed regularities
Reasoning and Proof: Evaluate conjectures

Learning Objectives:

Students will be able to see how prime numbers are used in a real life application. Students will also work through questions using their problem solving abilities.

Motivational Activities: Prime Numbers

It may be helpful to illustrate for students why some numbers are prime.

- There is a way to illustrate this with the NCTM: Illuminations Factorize Applet.
- For the applet, type a number on the right of the screen that you know is prime. Then, have students try to manipulate so the number of boxes (area) will equal that number. The only way they can accomplish this is by having a dimension of 1 on one of the sides, and the original number on the other.
- Now that students understand prime numbers, they can practice quickly recognizing common primes numbers by playing this one person game, Number Cop. In this game, students need to swerve the cop car to avoid the composite numbers while "hitting" the prime numbers.

Application Activities: RSA Cryptography

A not about the use of RSA Cryptography- RSA Cryptography is used to secure information on the Internet. This sort of public-key cryptography is what keeps passwords, signatures, credit card information, and other private information safe while on the Internet.

A few things on prime numbers:

- Sometimes it is difficult to decide if a number is prime or not. What are some ways that you decide if a number is prime? Is there a shorter way than checking if the number is divisible by every number before it?

- There have been some formulas to try to describe certain sets of primes. One of the many is: $n^2 - n + 41$ Evaluate this expression with n=0,1,2,3,4,5. Do you always get a prime number? CHALLENGE: Find an n less than 50 that does not generate a prime number.

RSA Cryptography

- The need: Keys needed to be kept secret for all pervious cryptosystems. This lead to a major trust issue.
- What is it? The receiver chooses the encryption key and decryption key. (This is contradictory to what has been done in the past, where the sender picked the key). The receiver then lists the encryption key in a directory (like a phonebook) so that anyone may use this key to send messages. The idea is that the receiver is the only one who knows how to decrypt the messages.
- How it is done:

  Need two prime numbers: p and q

  Need a number, e , which must be relatively prime to (p-1)x(q-1)

  n=p x q

  The encryption key (or public key) is the pair (n,e)

  Convert message into numbers (a=1, b=2, etc...) for each letter, the corresponding number becomes your variable, m.

  To get the coded number, use this formula: $C = m^e \bmod n$

- This is a little complicated, so let's go through an example.

  Pick two prime numbers, they are your p & q.

  Pick your e (keep in mind what it must be relatively prime to)

  Figure out your n

  Convert a message into numbers

  Go number by number, encoding each portion of the message

  Hand your encoded message to a friend, and see if they can figure out how to decode the message by working backwards. What information do they need in order to be able to decode the message?

- Why this is so cool

Imagine you have two REALLY huge prime numbers. This is going to make n a product of two really huge prime numbers. What does this mean? How easy will it be to break the code?

- Say that you know my n is 374626129. Can you figure out what my p and q are?

This is why we use RSA cryptography for extremely important information. Can you think of what RSA is used for? (Answer: encryption for the internet)

Sources Used:

The Cryptoclub

Number Cop: Hotmath.com

NCTM: Illuminations Factorize

Pattern Recognition

Target Audience:

Average middle school math students, grades 6-8.

National Standards (Grades 6-8):

Number and Operation: Work with fractions, decimals, and percents to solve problems
Number and Operation: Compare and order decimals
Algebra: Analyze and generalize a variety of patterns
Data Analysis and Probability: Collect data about a characteristic shared by two populations
Data Analysis and Probability: Use observations about differences between two or more samples to make conjectures about the populations involved
Reasoning and Proof: Examine patterns and structures to detect regularities
Reasoning and Proof: Formulate conjectures about observed regularities
Reasoning and Proof: Evaluate conjectures

Learning Objectives:

Students will be working on recognizing patterns to break codes. They will be actively engaged in problem solving to create and solve their own cryptograms.

Motivational Activities: Pattern Recognition

It is critical for students at this age to understand how math is essentially problem solving and critical thinking.

- One fun way for students to come to this realization is by having them frequently solve pattern games or pattern puzzles. These can of course be found all over the internet, but Casual Puzzles is one of my favorites.
- Another good game is the popular card game, Set. This game involves pattern recognition, sorting, and problem solving. Not to mention, it is fun!

Application Activities: Substitution Cipher

With a partner or small group, break this code.

20-15-4-1-25 25-15-21 23-9-12-12 12-5-1-18-14 1-2-15-21-20 3-15-4-5-19

Often times, substitution ciphers are cracked with a technique called frequency analysis (studying how commonly letters occur in the English language).

- On the table is a letter frequency chart. It may be beneficial to have students create their own from a large body of text as an additional activity

Analyze this cipher text using what you know from frequency analysis:

TFNRIUJ UZV DREP KZDVJ SVWFIV KYVZI UVRKYJ;
KYV MRCZREK EVMVI KRJKV FW UVRKY SLK FETV

Would have students work on this activity in groups. If they are having difficulty, ask leading questions about which letters appear most frequently in the ciphertext in comparison to the English language. Then use deduction by placement of letters to work out the rest of the message.

Caesar Cipher

- A Caesar Cipher is a special kind of coding system where the ciphertext is a "shifted" alphabet from the plaintext.
- This would be a place where students can look into the history of the Caesar Cipher and why it was used.
- Here is an example of a shift of 7 units. In this case, Caesar would write his message in ciphertext, then encode it, pass the message onto his generals and they would know the shift is 7 units.

Plaintext:	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Ciphertext	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G

On the table there are some Caesar Cipher wheels. Play with them and try to figure out how they work.

- Once you get the hang of them, try to crack these messages
  - ♣ YVTL DHZ UVA IBPSA PU H KHF.
  - ♣ YVTL PZ AOL NYLHALZA LTWPYL.
- Some questions to consider: Do we need to know the shift in order to crack these messages? How can we 'crack the code' if we don't know the shift? Are these safe ciphers to use in present day? Why or why not?

Challenge friends by writing your own secret messages.

The National Council of Teachers of Mathematics: Illuminations has a specific applet that allows students to quickly and see the shifting of letters, encoding, and decoding of messages. Codes Applet

Necessary Worksheets:

Letter Frequencies

Caesar Shifter Wheel

Sources Used:

National Council of Teachers of Mathematics: Illuminations

NCTM: Illuminations Codes Applet

Casual Puzzles

Set Online

The Cryptoclub

Made on a Mac

Modular Numbers

Target Audience:

Average middle school math students, grades 6-8.

National Standards (Grades 6-8):

Number and Operation: Use negative integers
Algebra: Relate and compare different forms of representation of a relationship
Algebra: Model and solve contextualized problems using tables

Learning Objectives:

Students will use modular arithmetic to explore cryptography. The use of modular arithmetic will develop problem solving skills and number sense.

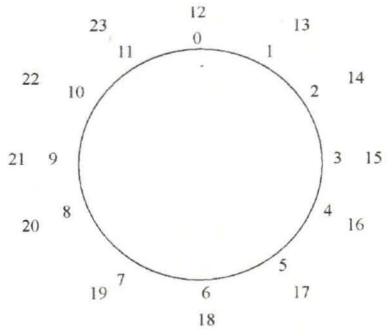
Motivational Activities: Modular Numbers

Mancala: Mancala Online

- Mancala is a classic game where players take turns moving their stones around the board and trying to get as many stones in their "pot" (Link to the instructions of Mancala)
- This game uses modular arithmetic without students knowing they are using it.
- This uses modular arithmetic because in order to play with strategy, students must think about where their stones are going to land on their side, in their Mancala (or "pot"), on their opponent's side, or in their opponent's Mancala.

"Clock" Arithmetic

- It is 4:00 PM, what time is it in military time?
- It is 21:30, what time is it in a 12-hour clock?
- Congratulations! You just successfully did modular arithmetic.
- Notice in the below diagram that the numbers 0, 12, and 24 are in the same position. What are five numbers that would fall in the 8 position?



Change the situation

- What if it is a 10-hour clock? What is 12:00 in a 10-hour clock?
- A 4 hour-clock? What is 9:00 is a 4-hour clock?
- Here is a link to an online applet that demonstrates "clock arithmetic"
  Interactive Clock Arithmetic Applet

Application Activity: Modular Arithmetic Coding

- On the table are cipher wheels. Use them to change letters into numbers. Think of a=0, b=1, c=2, etc....
- Encrypt "cryptography" using the times-3 cipher as the table describes

Times -3 Cipher	C	R	Y	P	T	O	G	R	A	P	H	Y
Letter-Number	2	17										
Multiply by 3	6	51										
Reduce mod 26	6	25										
Number-Letter	G	Z										

Make your own times-x cipher table to encode a message. (x being any number you wish)

- If you are feeling really crazy, try making x negative.

Questions

- Is this cipher easy or difficult to crack? What information do you need in order to decrypt? How would one go about decrypting with no information?

Sources Used:

The Cryptoclub

Mancala Online: Rocketsnail Games

Mancala Instructions: Pressman Toy

Interactive Clock Arithmetic Applet

Links and Resources

Below are a list of links to various middle school math online activity sites followed by a short description.

Quia Shared Activities

This website contains almost 1,700 activities for mathematics. You can search specific topics, or browse through all of the activities the site has to offer. There are different categories of activities depending on what you are looking to do in the classroom. A key to the different symbols can be found here. These activities are all online, but many of them can easily be converted into simple games for the classroom that don't require a computer or the internet.

Illuminations: Resources for Teaching Math

This website, set up by the National Council of Teachers of Mathematics has pre-made activities and lessons that align with the national mathematics standards. Both the activities and lessons can be searched for specific topics and concepts.

Math.com: The World of Math Online

This website goes through tutorials and brief notes and examples for topics in the subjects of pre-algebra, algebra, and geometry.

Math Playground

This site has games, word problems, logic puzzles, and math videos and that can be accessed through the toolbar on the left of the page. This site provides resources for elementary and middle school mathematics. Most of the activities and games on this site are online and would require a computer for groups of students in the class. However, some of the games can be adapted to be done with tangible items instead of online.

National Library of Virtual Manipulatives

This site contains virtual manipulatives for algebra grades 6-8. Again, all of these manipulatives are online, however, many can be adapted to a more tangible activity is computers are not available.

Shodor Interactivate Activities

This website has online activities covering: number and operations, geometry, algebra, probability, statistics, modeling, and discrete. These activities can be navigated through by topic and there is a brief description of the activity to the right of the link. Once again, all of these activities are digital, but some can be adopted to not require the internet or a computer.